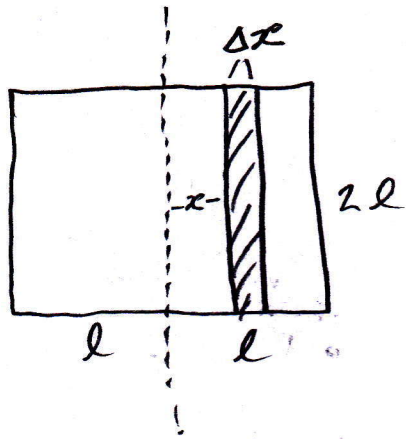
2011 Q8.

(a)



$$\rho = \frac{m}{\text{area}} \Rightarrow \text{mass} = \rho(\text{Area}) = \rho(2l)(2l$$
$$= \rho 4l^2$$

Mass of each strip $= \Delta m = \rho(\text{Area})$
$$= \rho(2l)(\Delta x)$$
$$= 2l\rho\Delta x$$

$$I = \sum \Delta m r^2$$
$$= \sum (2l\rho\Delta x)x^2$$
$$= \sum 2l\rho x^2 \Delta x$$
$$= \int_{-l}^{l} 2l\rho x^2 dx$$
$$= \left[2l\rho \frac{x^3}{3}\right]_{-l}^{l}$$

$$= \left(2l\rho\frac{l^3}{3}\right) - \left(2l\rho\frac{(-l)^3}{3}\right)$$

$$= 2l\rho\frac{l^3}{3} + 2l\rho\frac{l^3}{3}$$

$$= 4\ell\rho\frac{\ell^4}{3}$$

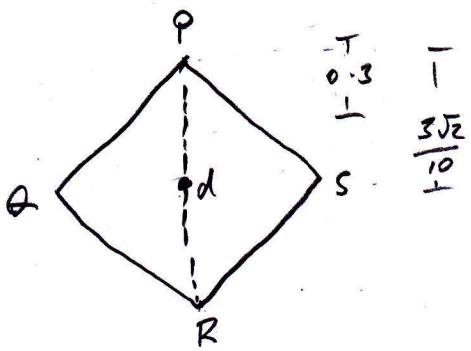
$$= (4\ell^2\rho)\frac{\ell^2}{3}$$

$$= m\frac{\ell^2}{3} \qquad [\text{since } m = 4\ell^2\rho]$$

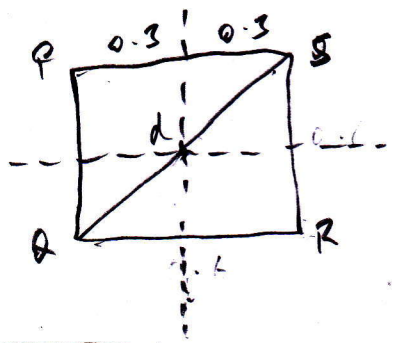
$$= \frac{1}{3}m\ell^2$$

(b) (i) Gain in KE = Loss in PE

$$\frac{1}{2}I\omega^2 = mgh$$



We need to work out I:



$$I_d = I_x + I_y$$

$$= \frac{1}{3}(0.3)^2m + \frac{1}{3}(0.3)^2m$$

$$= \frac{2}{3}m(0.3)^2$$

$$\sqrt{(0.6)^2+(0.6)^2} = |QS|$$

$$\frac{3\sqrt{2}}{5} = |QS| \Rightarrow |dP| = \frac{3\sqrt{2}}{10}$$

$$\Rightarrow I_P = \frac{2}{3}m(0.3)^2 + m\left(\frac{3\sqrt{2}}{10}\right)^2 \qquad [\text{i.e. } I_P = I_d + m|Pd|^2]$$

$$\Rightarrow \frac{1}{2}\left[\frac{2}{3}m(0.3)^2 + m\left(\frac{3\sqrt{2}}{10}\right)^2\right]\omega^2 = mg\left(\frac{3\sqrt{2}}{10} - 0.3\right)$$

$$\Rightarrow \omega^2 = \frac{2g\left(\frac{3\sqrt{2}}{10} - 0.3\right)}{\frac{3}{50} + \frac{18}{100}} \Rightarrow \omega^2 = 10.1682$$

$$\Rightarrow \omega = 3.19 \text{ rad s}^{-1}$$

Note: To work out I we can also use $\frac{4}{3}ml^2$

$$\Rightarrow I = \frac{4}{3}m(0.3)^2 + \frac{4}{3}m(0.3)^2 \quad \text{etc.}$$

(ii) $I_{system} = I_{lamina} + I_{mass}$

$$= \left(\frac{2}{3}m(0.3)^2 + m\left(\frac{3\sqrt{2}}{10}\right)^2\right) + m\left(\frac{3\sqrt{2}}{5}\right)^2$$

$$= \frac{3}{50}m + \frac{18}{100}m + \frac{18}{25}m$$

$$= 0.96m$$

$$T = 2\pi\sqrt{\frac{0.96m}{Mgh}}$$

$$\left[Mgh = \left(\frac{3\sqrt{2}}{10}\right)mg + \frac{3\sqrt{2}}{5}mg = 0.9\sqrt{2}mg\right]$$

$$= 2\pi\sqrt{\frac{0.96m}{0.9\sqrt{2}mg}}$$

$$= 1.74 \text{ sec.}$$

$$2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.96m}{0.9\sqrt{2}mg}}$$

$$\Rightarrow L = \frac{0.96}{0.9\sqrt{2}} = 0.75m$$