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EXAMPLE 2

In a by-election, a random sample of 900 voters suggests that 35% will vote for candidate A.

(i) Find the standard error at the 95% confidence level.

(ii) If a random sample of 400 voters was used, would the standard error (at the 95% confidence level) be greater than or less than 1·6%? Justify your answer.

(iii) Candidate A is a billionaire who wishes to determine his level of support to within $\pm\frac{1}{2}\%$. What sample size should be taken at the 95% confidence level?

Solution

(i) Standard error $= \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0{\cdot}35)(1-0{\cdot}35)}{900}} = 0{\cdot}016 = 1{\cdot}6\%$

(ii) Since the sample size, $n = 400$, is less than the sample size $n = 900$ from part **(i)**, we state:

Method 1

Standard error $= \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0{\cdot}35)(0{\cdot}65)}{400}} = 0{\cdot}024 = 2{\cdot}4\%$

Hence, standard error is greater than 1·6%.

Method 2

Since $n = 900$ in part **(i)** is greater than $n = 400$ in part **(ii)**, this means the answer in part **(i)** is more accurate than the answer to part **(ii)**.

$\therefore$ $n = 400$ has a standard error $> 1{\cdot}6\%$.

(iii) Note: To within $\pm\frac{1}{2}\%$ means that the margin of error should be $\frac{1}{2}\%$.

Let n = size of the random sample

$$E = \frac{1}{2}\% = 0{\cdot}005$$

$$\hat{p} = 0{\cdot}35$$

Then $E = 1{\cdot}96 = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

becomes $0{\cdot}005 = 1{\cdot}96\sqrt{\frac{(0{\cdot}35)(0{\cdot}65)}{n}}$

$$0{\cdot}000025 = 3{\cdot}8416\frac{0{\cdot}2275}{n} \quad \text{(square both sides)}$$

$$0{\cdot}000025n = 0{\cdot}873964$$

$$n = 34{,}959$$

A sample size of 34,959 would be required to obtain a margin of error of $\frac{1}{2}\%$.

Some notes on margin of error

- On our course, the margin of error is **always** at the 95% level of confidence.
- As the sample size increases the margin of error decreases.
- At the 95% level of confidence a sample of about
  - (i) 80 has a margin of error approximately ±11%
  - (ii) 1,000 has a margin of error approximately ±3·2%.
- The size of the (original) population does not matter.
- If the sample size, m, is doubled (say 500 to 1,000) the margin of error, E, is **not** halved.
- The margin of error estimates how accurately the results of a poll reflect the true feelings of the population.