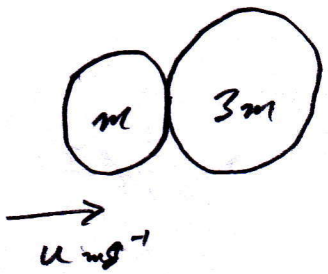
2010 Q5.

(a) (i)



(i) PCM: $m(u) + 3m(0) = mV_1 + 3mV_2$

$\Rightarrow \quad u = V_1 + 3V_2 \quad ①$

NEL: $V_1 - V_2 = -e(u-0)$

$\Rightarrow V_1 - V_2 = -ue \quad ②$

① $V_1 = u - 3V_2$ and $V_2 = \frac{u - V_1}{3}$

Substituting into ②

$\Rightarrow V_1 - \left(\frac{u-V_1}{3}\right) = u - ue$

$\Rightarrow 3V_1 - (u - V_1) = u - 3ue$

$\Rightarrow 4V_1 - u = -3ue$

$\Rightarrow \quad V_1 = \frac{u - 3ue}{4}$

$\Rightarrow \quad V_1 = \frac{u(1-3e)}{4}$

Similarly: $V_2 = \frac{u(1+e)}{4}$

(ii)

KE before $= \frac{1}{2}mu^2$

KE After $= \frac{1}{2}mV_1^2 + \frac{1}{2}(3m)V_2^2$

$$= \frac{1}{2}m\left(\frac{u(1-3(\frac{1}{4}))}{4}\right)^2 + \frac{1}{2}(3m)\left(\frac{u(1+\frac{1}{4})}{4}\right)^2$$

$$= \frac{1}{2}m\left(\frac{u\,\frac{1}{4}}{4}\right)^2 + \frac{3}{2}m\left(\frac{\frac{5}{4}u}{4}\right)^2$$

$$= \frac{1}{2}m\left(\frac{u^2}{256}\right) + \frac{3}{2}m\left(\frac{25u^2}{256}\right)$$

$$= \frac{mu^2}{512} + \frac{75mu^2}{512}$$

$$= \frac{76mu^2}{512} = \frac{19mu^2}{128}$$

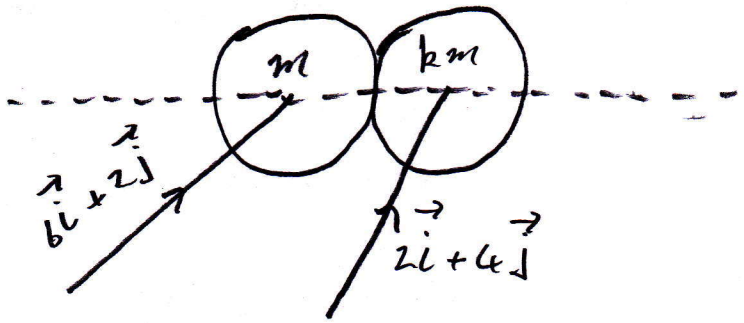
$\Rightarrow$ Loss in KE $= \frac{1}{2}mu^2 - \frac{19}{128}mu^2$

$$= \frac{45}{128}mu^2$$

% Loss in KE $= \dfrac{\frac{45}{128}mu^2}{\frac{1}{2}mu^2} \times 100$

$$= 70.3\,\%$$

(b) (i)



PCM: $m(6) + km(2) = mV_1 + kmV_2$

$\Rightarrow 6 + 2k = V_1 + kV_2$

$\Rightarrow V_1 = 6 + 2k - kV_2 \Rightarrow V_1 = 6 + k(2 - V_2)$

and $kV_2 = 6 + 2k - V_1 \Rightarrow V_2 = \dfrac{6 + 2k - V_1}{k}$ ①

NEL: $V_1 - V_2 = -e(6 - 2) \Rightarrow V_1 - V_2 = -4e$ ②

Substituting ① into ②

$\Rightarrow V_1 - \left(\dfrac{6 + 2k - V_1}{k}\right) = -4e$

$\Rightarrow kV_1 - 6 - 2k + V_1 = -4ke$

$\Rightarrow V_1(k + 1) = -4ke + 6 + 2k$

$\Rightarrow V_1 = \dfrac{6 + 2k - 4ek}{k + 1}$

Similarly: $V_2 = \dfrac{6 + 4e + 2k}{k + 1}$

Parallel directions $\Rightarrow$ slopes are equal

Note: $\vec{j}$ directions are unchanged after impact

$$\Rightarrow \quad \frac{2}{V_1} = \frac{4}{V_2}$$

$$\Rightarrow \quad \frac{2}{\frac{6+2k-4ek}{k+1}} = \frac{4}{\frac{6+4e+2k}{k+1}}$$

$$\Rightarrow \quad \frac{2(k+1)}{6+2k-4ek} = \frac{4(k+1)}{6+4e+2k}$$

$$\Rightarrow \quad \frac{1}{6+2k-4ek} = \frac{2}{6+4e+2k} \qquad \left[\div \text{ by } 2(k+1)\right.$$

$$\Rightarrow \quad 6+4e+2k = 12+4k-8ek$$

$$\Rightarrow \quad 4e+8ek = 12+4k-6-2k$$

$$\Rightarrow \quad e(4+8k) = 6+2k$$

$$\Rightarrow \quad e = \frac{6+2k}{4+8k}$$

$$\Rightarrow \quad e = \frac{3+k}{2+4k}$$

(ii) $e \leq 1 \Rightarrow \frac{3+k}{2+4k} \leq 1 \Rightarrow 3+k \leq 2+4k \Rightarrow 1 \leq 3k$

$$\Rightarrow \frac{1}{3} \leq k .$$